Math 2326, Test III

Name ______________________________

1. Find the general solution:
   $w'' - 16w = e^{-4t}$

   answer: $w(t) = C_1 e^{4t} + C_2 e^{-4t} - \frac{1}{8} t e^{-4t}$

2. Consider the RLC circuit problem:
   $Lq'' + Rq' + \frac{1}{C} q = sin(\omega t)$
   where $L = C = 1, R = 0.1$. Assume a particular solution of the form $\alpha sin(\omega t) + \beta cos(\omega t)$, plug this into the equation and write a system of two equations:
   $a_{11}\alpha + a_{12}\beta = b_1$
   $a_{21}\alpha + a_{22}\beta = b_2$
   Now find the determinant $det(\omega) = a_{11}a_{22} - a_{21}a_{12}$ of the matrix of this linear system (note that it is always positive). Finally, find the value of ω which minimizes $det(\omega)$, by taking the derivative of $det(\omega)$ and setting it to zero. This will be the resonant frequency of this circuit, that is, the frequency which produces the largest amplitude $\alpha^2 + \beta^2$.

   answer: $det(\omega) = (1 - \omega^2)^2 + 0.01\omega^2$
   $\omega_{min} = 0.9975$

3. Find all 4 equilibrium points of the nonlinear system:

$$\frac{dx}{dt} = x(2 - x - y)$$
$$\frac{dy}{dt} = y(3 - 2x - y)$$

and classify ONE of them as a source, sink, saddle point, center, spiral source or spiral sink.

answer: $(0,0)$ is source, $(2,0),(0,3)$ are sinks, $(1,1)$ is saddle.

4. Find the Laplace transform of the solution to:
$y'' + 5y' + 4y = 3sin(t)$, with $y(0) = 1, y'(0) = 0$

answer: $L(y) = [\frac{3}{s^2+1} + s + 5]/[s^2 + 5s + 4]$

5. Solve, **using Laplace transforms**
$y' + 2y = 4e^{2t}$, with $y(0) = 0$.

answer: $y(t) = e^{2t} - e^{-2t}$