

### Math 2326, Test III

Name \_\_\_\_\_

1. Find the solution to:

$$w'' + 6w' + 8w = \sin(t)$$

$$w(0) = -1$$

$$w'(0) = 1$$

answer:  $w(t) = -\frac{119}{85}e^{-2t} + \frac{40}{85}e^{-4t} + \frac{7}{85}\sin(t) - \frac{6}{85}\cos(t)$

2. Find the general solution to:

$$y'' + 4y = 3t + 2$$

answer:  $y(t) = C_1\sin(2t) + C_2\cos(2t) + \frac{3}{4}t + \frac{1}{2}$

3. Find all equilibrium points of the nonlinear system:

$$\begin{aligned}\frac{dx}{dt} &= x^2 - y^2 \\ \frac{dy}{dt} &= xy - 9\end{aligned}$$

and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

answer:  $(3, 3)$  is spiral source,  $(-3, -3)$  is spiral sink

4. Find the Laplace transform of the solution to:  
 $y'' + 6y' + 8y = \sin(t)$ , with  $y(0) = -1, y'(0) = 1$

answer:  $L(y) = \frac{1}{(s^2+1)(s+2)(s+4)} - \frac{s+5}{(s+2)(s+4)}$

5. Solve **using Laplace transforms**:  
 $y'' - 4y = 24$ , with  $y(0) = 0, y'(0) = 0$

answer:  $y(t) = -6 + 3e^{2t} + 3e^{-2t}$