Math 2326, Test III

Name ____________________________

1. Find the general solution:
$w'' + 2w' + w = e^{-t}$

answer: $w(t) = C_1 e^{-t} + C_2 t e^{-t} + \frac{1}{2} t^2 e^{-t}$

2. Find any 5 of the 6 equilibrium points of the nonlinear system:

$\frac{dx}{dt} = -8x^2 - 6xy + 480x$
$\frac{dy}{dt} = -x^2 y - y^3 + 2500y$

and classify any two of them as a source, sink, saddle point, spiral source, spiral sink, or center.

answer: $(0,0)$ is source, $(0,50)$ is saddle, $(0,-50)$ is saddle,
$(60,0)$ is sink, $(30,40)$ is sink, $(46.8, 17.6)$ is saddle.

3. a. Find the general solution of the undamped spring problem, with a periodic applied force field: $my'' + ky = cos(\omega t)$

answer: $y(t) = C_1 cos(\sqrt{\frac{k}{m}}t) + C_2 sin(\sqrt{\frac{k}{m}}t) + \frac{1}{k-\omega^2 m} cos(\omega t)$

b. Near what value of the frequency ω will a very large oscillation of the spring result?

answer: $\omega = \sqrt{\frac{k}{m}}$

4. Find the Laplace transform of the solution to problem 1, with initial conditions $w(0) = 2, w'(0) = 3$. (Don't need to find $w(t)$.)

answer: $L(w) = \frac{1}{(s+1)^3} + \frac{2s+7}{(s+1)^2}$

5. Find the inverse Laplace transform of $F(s) = \frac{2s+3}{s^2+s+1}$

answer: $f(t) = 2e^{-t/2} cos(\sqrt{3}t/2) + \frac{4}{\sqrt{3}} e^{-t/2} sin(\sqrt{3}t/2)$

6. Find the Laplace transform of the solution of $y'' - 3y' + 5y = g(t)$, with $y(0) = 0, y'(0) = 0$, where $g(t) = 0$ for $t < 2$ and $g(t) = e^{2t}$ for $t \geq 2$.

answer: $L(y) = \frac{e^4 e^{-2s}}{(s-2)(s^2-3s+5)}$