

Math 2326, Test III

Name _____

1. Find the general solution:

$$w'' + 4w' + 4w = 25\sin(t)$$

answer: $w(t) = C_1e^{-2t} + C_2te^{-2t} + 3\sin(t) - 4\cos(t)$

2. Find the equilibrium point for which neither x nor y is 0, of the non-linear system:

$$\begin{aligned}\frac{dx}{dt} &= x(100 - x - 2y) \\ \frac{dy}{dt} &= y(150 - x - 4y)\end{aligned}$$

and classify it as a source, sink, saddle point, spiral source, spiral sink, or center.

answer: $(50, 25)$ is a sink, because Jacobian eigenvalues are $-19, -131$.

3. Find the Laplace transform of the solution to:
 $y'' + 4y' + 4y = 25\sin(t)$, with $y(0) = 2, y'(0) = 3$

answer: $L(y) = \frac{25}{(s^2+1)(s+2)^2} + \frac{2s+11}{(s+2)^2}$

4. Find the inverse Laplace transform of $F(s) = \frac{e^{-3s}}{(s^2+25)}$

answer: $f(t) = \frac{1}{5}u_3(t)\sin(5(t-3))$

5. Solve **using Laplace transforms**:
 $y'' - 4y = 24$, with $y(0) = 0, y'(0) = 0$

answer: $y(t) = -6 + 3e^{2t} + 3e^{-2t}$