

Math 2326, Test III

Name _____

1. Solve $w'' + 12w' + 36w = e^{3t}$, with $w(0) = 0, w'(0) = \frac{2}{9}$.

answer: $w(t) = -\frac{1}{81}e^{-6t} + \frac{1}{9}te^{-6t} + \frac{1}{81}e^{3t}$

2. Find the general solution of $y'' - 2y' + 7y = 28t + 6$

answer: $y(t) = C_1e^t\cos(\sqrt{6}t) + C_2e^t\sin(\sqrt{6}t) + 4t + 2$

3. Find the inverse Laplace transform of $F(s) = \frac{e^{-3s}}{s^2+1}$

answer: $f(t) = u_3(t)\sin(t - 3)$

4. The nonlinear system:

$$\begin{aligned}\frac{dx}{dt} &= x(-x - 3y + 150) \\ \frac{dy}{dt} &= y(-2x - y + 100)\end{aligned}$$

has 4 equilibrium points. $(0,100)$ is one of the equilibrium points, classify it as a source, sink, saddle point, spiral source, spiral sink or center, and find the other three equilibrium points.

answer: $(0, 0)$, $(0, 100)$, $(150, 0)$, $(30, 40)$; $(0, 100)$ is a sink

5. Solve **using Laplace transforms**:

$$y'' - 4y = 12, \text{ with } y(0) = 0, y'(0) = 0$$

answer: $y(t) = -3 + \frac{3}{2}e^{2t} + \frac{3}{2}e^{-2t}$