

Math 2326, Test III

Name _____

1. Find the general solution of $w'' - 5w' + 4w = 8t + 2$

answer: $w(t) = C_1e^{4t} + C_2e^t + 2t + 3$

2. Find the solution of $y'' - 2y' + 10y = 0$, with $y(0) = 1, y'(0) = -5$.

answer: $y(t) = e^t[\cos(3t) - 2 \sin(3t)]$

3. Find all 4 of the equilibrium points of the nonlinear system:

$$\begin{aligned}\frac{dx}{dt} &= 20x - x^2 - xy \\ \frac{dy}{dt} &= 30y - y^2 - 2xy\end{aligned}$$

and classify any two of them as a source, sink, saddle point, spiral source, spiral sink, or center.

answer: $(0, 0)$ is source, $(0, 30)$ is sink, $(20, 0)$ is sink, $(10, 10)$ is saddle

4. Solve, using Laplace transforms

$$y'' - 5y' + 4y = e^{2t}, \text{ with } y(0) = 0, y'(0) = 0$$

$$\text{answer: } y(t) = \frac{1}{6}e^{4t} + \frac{1}{3}e^t - \frac{1}{2}e^{2t}$$

5. Find the inverse Laplace transform of $F(s) = \frac{1}{s^2 - 4s + 12}$

$$\text{answer: } f(t) = \frac{1}{\sqrt{8}}e^{2t}\sin(\sqrt{8}t)$$

6. Find the Laplace transform of the solution to:

$$y'' = u_{2\pi}(t)\sin(2t), \text{ with } y(0) = 1, y'(0) = 2.$$

$$\text{answer: } L(y) = \frac{s+2}{s^2} + e^{-2\pi s} \frac{2}{s^2(s^2+4)}$$