

Math 2326, Test III

Name _____

1. Find the general solution:

$$w'' + 8w' + 25w = e^t$$

answer: $w(t) = C_1 e^{-4t} \cos(3t) + C_2 e^{-4t} \sin(3t) + \frac{1}{34} e^t$

2. Find all 5 of the equilibrium points of the nonlinear system:

$$\begin{aligned} \frac{dx}{dt} &= 25x - 3x^2 - 4xy \\ \frac{dy}{dt} &= 25y - x^2y - y^3 \end{aligned}$$

and classify any two of them as a source, sink, saddle point, spiral source, spiral sink, or center.

answer: $(0, 0)$ is source, $(0, 5)$ is saddle, $(0, -5)$ is saddle, $(25/3, 0)$ is sink, $(3, 4)$ has eigenvalues 0, -41.

3. Find the Laplace transform of the solution to:
 $y'' + 5y' + 4y = \sin(t)$, with $y(0) = 0, y'(0) = 5$

answer: $L(y) = \frac{1}{(s^2+1)(s^2+5s+4)} + \frac{5}{s^2+5s+4}$

4. Find the inverse Laplace transform of $F(s) = \frac{e^{-3s}}{s^2+1}$

answer: $f(t) = u_3(t)\sin(t - 3)$

5. Solve **using Laplace transforms**:
 $y'' - 4y = 12$, with $y(0) = 0, y'(0) = 0$

answer: $y(t) = -3 + \frac{3}{2}e^{2t} + \frac{3}{2}e^{-2t}$