Math 2326, Final Exam

Name ______________________________

1. Reduce the second order equation $y''(t) - 5y'(t) + 6y(t) = 0, y(0) = 1, y'(0) = 1$ to a system of first order equations ($x' = Ax, x(0) = x_0$, where $x(t)$ is a vector, A is a 2 by 2 matrix).

answer:

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with $x_1(0) = 1, x_2(0) = 1$

2. Solve the system $x' = Ax, x(0) = x_0$ of problem 1, using the techniques of chapter 3, that is, by finding the eigenvalues and eigenvectors of A, and writing the general solution for $x(t)$, then finding the values for the arbitrary constants which make $x(0) = x_0$.

answer: $x(t) = e^{2t} \begin{bmatrix} 2 \\ 4 \end{bmatrix} + e^{3t} \begin{bmatrix} -1 \\ -3 \end{bmatrix}$

3. Now solve the second order equation of problem 1 (with initial conditions) using Laplace transforms. (Hint: $L(e^{at}) = \frac{1}{s-a}$, and $L(y') = sL(y) - y(0)$.)

answer: $y(t) = 2e^{2t} - e^{3t}$

4. Now solve the second order equation of problem 1 (with initial conditions) by trying solutions of the form $y(t) = e^{rt}$ and determining the values of r (the roots of the characteristic polynomial) that make this a solution.

answer: $y(t) = 2e^{2t} - e^{3t}$

5. Solve $y' = e^{y+t}$, with $y(0) = ln(\frac{1}{2})$.

answer: $y(t) = -ln(3 - e^t)$

6. Find the general solution, using any technique you prefer, of $y'' + 6y' + 9y = 2cos(3t)$.

answer: $y(t) = C_1 e^{-3t} + C_2 t e^{-3t} + \frac{1}{9} sin(3t)$