Math 2326, Final Exam

Name ____________________________

Work any 5 problems. Indicate clearly which problem NOT to be graded.

1. For what range of initial values (A) will the solution of

   $y' = (y-2)^3(8-y)(1+y)$, with $y(0) = A$

   converge (as $t \to \infty$) to $y = 2$? For what range will the solution converge to $y = 8$? (Hint: construct the phase line.)

   answer: for $A = 2, y \to 2$, for $A > 2, y \to 8$

2. If the eigenvalues (λ_i) and eigenvectors (z_i) of the 3 by 3 matrix A are:

   $\lambda_1 = 4.5, z_1 = (1,3,-2); \lambda_2 = -3, z_2 = (2,0,1); \lambda_3 = 7, z_3 = (0,0,1)$

   write the general solution of $y' = Ay$, where $y = (y_1, y_2, y_3)$.

   answer: $y = C_1 e^{4.5t} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} + C_2 e^{-3t} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + C_3 e^{7t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3. **Use Laplace transforms** to solve $y'' + 9y' + 18y = 54e^{3t}$, with $y(0) = 0, y'(0) = 0$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = -3e^{-3t} + 2e^{-6t} + e^{3t}$

4. Verify that $(30, 40)$ is an equilibrium point of the system below, and classify it as a source, sink, saddle point, spiral source, spiral sink, or center.

$$\frac{dx}{dt} = -8x^2 - 6xy + 480x$$
$$\frac{dy}{dt} = -x^2y - y^3 + 2500y$$

answer: eigenvalues of Jacobian are $-101, -3339$, so $(30, 40)$ is a sink

5. Solve the differential equation of problem 3 **without** using Laplace transforms.

answer: $y(t) = -3e^{-3t} + 2e^{-6t} + e^{3t}$

6. A wine chilled at 10^oC is suddenly moved to a room where the temperature is 30^oC. If at $t = 0$ the wine is warming up at the rate of $5^oC/hour$, when will the wine reach 15^oC? Assume Newton's law of cooling, that is, $\frac{dT}{dt} = -k(T - T_0)$, where T is temperature, t is time, k is a constant, and T_0 is the room temperature.

answer: $t = 1.150$ hours