

Math 2326, Final Exam

Name _____

1. a. Find the solution to the following system, with $x(0) = 2, y(0) = 8$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = e^{2t} \begin{bmatrix} 8 \\ 0 \end{bmatrix} + e^{-2t} \begin{bmatrix} -6 \\ 8 \end{bmatrix}$

- b. Find all equilibrium points of problem 1a, and classify each as a source, sink or saddle point.

answer: $(0, 0)$ is a saddle

2. Solve $y'' + 25y = 5\delta_3(t)$, $y(0) = 0$, $y'(0) = 0$ using Laplace transforms.
(Hint: $L(\delta_a(t)) = e^{-as}$, $L(u_a(t)q(t-a)) = e^{-as}Q(s)$, $L(\sin(\omega t)) = \frac{\omega}{s^2 + \omega^2}$
and $L(y'') = s^2L(y) - sy(0) - y'(0)$.)

answer: $y(t) = u_3(t)\sin(5(t-3))$

3. Find the general solution of $y' = e^{y+t}$.

answer: $y(t) = -\ln(C - e^t)$

4. Find the general solution of $y'' - 6y' - 7y = e^{7t}$.

answer: $y(t) = C_1e^{7t} + C_2e^{-t} + \frac{1}{8}te^{7t}$

5. Find all equilibrium points of $y' = y^2 - 6y - 7$, and classify each as a source, sink or node. What happens to $y(t)$ as $t \rightarrow \infty$, if $y(0) = 0$?

answer: $y = 7$ is source, $y = -1$ is sink. $y(t) \rightarrow -1$ as $t \rightarrow \infty$

6. Consider the spring problem $my'' + by' + ky = 0$, where $m = 1, k = 13$. If b (friction coefficient) is below a certain limit, $y(t)$ will oscillate as it decays to 0; if it is above this limit, $y(t)$ will decay directly to 0. What is this limiting value of b ?

answer: $b = \sqrt{52}$