Math 2326, Final Exam

Name ____________________________

Work any 5 problems. Indicate clearly which problem NOT to be graded.

1. Find all equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

   $\frac{dx}{dt} = x^2 - y^2$
   $\frac{dy}{dt} = xy - 4$

   answer: $(2,2)$ is spiral source, $(-2,-2)$ is spiral sink

2. Find the general (real) solution of

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{3t} \begin{bmatrix} 4cos(\sqrt{7}t) \\ cos(\sqrt{7}t) + \sqrt{7}sin(\sqrt{7}t) \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 4sin(\sqrt{7}t) \\ sin(\sqrt{7}t) - \sqrt{7}cos(\sqrt{7}t) \end{bmatrix}$

3. **Use Laplace transforms** to solve $y' + y = e^{-2t}$, with $y(0) = 2$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

   answer: $y(t) = 3e^{-t} - e^{-2t}$

4. Solve the differential equation of problem 3 **without** using Laplace transforms.

   answer: $y(t) = 3e^{-t} - e^{-2t}$

5. Solve the following partially decoupled **nonlinear** system:

$$x' = -3t^2xy \qquad x(1) = 6$$
$$y' = 3t^2y^2 \qquad y(1) = 1$$

answer: $x(t) = 6(2 - t^3), y(t) = \frac{1}{2-t^3}$

6. If a harmonic oscillator with mass m and spring constant k is subject to an external force $sin(\omega t)$ the height of the mass $y(t)$ satisfies the equation:

   $my'' + ky = sin(\omega t)$

   Find the general solution of this equation. What value of ω will produce a very large oscillation? (In solving the above equation, you can assume ω is not exactly equal to this "resonant" frequency.)

   answer: $y(t) = C_1 sin(\sqrt{\frac{k}{m}}t) + C_2 cos(\sqrt{\frac{k}{m}}t) + \frac{1}{k-m\omega^2} sin(\omega t)$
   $\omega = \sqrt{\frac{k}{m}}$ will produce large oscillation