

## Math 2326, Final Exam

Name \_\_\_\_\_

Solve any 5 of the problems. Clearly mark which one NOT to grade.

1. a. Find the general solution to the following system.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\text{answer: } \begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-6t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

- b. Find all equilibrium points of problem 1a, and classify each as a source, sink, saddle, center, spiral sink or spiral source.

answer:  $(0, 0)$  is a sink.

2. Solve  $y'' + y = \delta_3(t)$ ,  $y(0) = 0$ ,  $y'(0) = 0$  using Laplace transforms.  
(Hint:  $L(\delta_a(t)) = e^{-as}$ ,  $L(u_a(t)q(t-a)) = e^{-as}Q(s)$ ,  $L(\sin(\omega t)) = \frac{\omega}{s^2 + \omega^2}$   
and  $L(y'') = s^2 L(y) - sy(0) - y'(0)$ .)

answer:  $y(t) = u_3(t)\sin(t - 3)$

3. Find the solution of  $y' = e^{2y-t}$  with  $y(0) = 0$ .

answer:  $y(t) = -\frac{1}{2}\ln(2e^{-t} - 1)$

4. a. Find all three equilibrium points of the predator-prey system:

$$\begin{aligned}x' &= x\left(2 - \frac{2}{3}x - y\right) \\y' &= y(-2 + 4x)\end{aligned}$$

answer:  $(0, 0), (3, 0), \left(\frac{1}{2}, \frac{5}{3}\right)$

b. Is  $x$  or  $y$  the predator population? Why?

answer:  $y$ . When  $x = 0, y \rightarrow 0$ .

c. Only one equilibrium point has both  $x$  and  $y$  greater than zero. Classify it as a sink, source, saddle, spiral sink, spiral source or center.

answer:  $\left(\frac{1}{2}, \frac{5}{3}\right)$  is spiral sink.

5. Consider the spring problem  $my'' + by' + ky = 0$ , where  $m = 1, k = 10$ . The friction coefficient  $b$  cannot be negative, for physical reasons; what would happen to the solution as  $t \rightarrow \infty$ , if it were negative? If  $b > 0$  is below a certain limit,  $y(t)$  will oscillate as it decays to 0; if it is above this limit,  $y(t)$  will decay directly to 0. What is this limiting value of  $b$ ?

answer: If  $b < 0$ , solution goes to infinity with time.  $b_{limit} = \sqrt{40}$

6. Find the solution of  $y'' + 6y' + 25y = e^{2t}$  with  $y(0) = 0, y'(0) = 1$ .

answer:  $y(t) = -\frac{1}{41}e^{-3t}\cos(4t) + \frac{9}{41}e^{-3t}\sin(4t) + \frac{1}{41}e^{2t}$