Math 2326, Final Exam

Name ____________________________

Work any 5 problems. Indicate clearly which problem NOT to be graded.

1. For what range of initial values (A) will the solution of

   $y' = (y-3)^3(5-y)(2+y)$, with $y(0) = A$

   converge (as $t \to \infty$) to $y = 5$?

   answer: $A > 3$

2. Find the solution of $y' = e^{-2y+t}$ with $y(0) = 0$.

   answer: $y(t) = \frac{1}{2} ln(2e^t - 1)$

3. a. If the eigenvalues (λ_i) and eigenvectors (z_i) of the 3 by 3 matrix A are:

$\lambda_1 = -4, z_1 = (1,-3,1); \lambda_2 = -3, z_2 = (-2,0,3); \lambda_3 = -7, z_3 = (0,1,0)$

write the general solution of $y' = Ay$, where $y = (y_1, y_2, y_3)$.

answer: $y = C_1 e^{-4t} \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} + C_2 e^{-3t} \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} + C_3 e^{-7t} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

b. $y = (0,0,0)$ will be an equilibrium point for $y' = Ay$ no matter what the 3 by 3 matrix A is. Classify this equilibrium point as a source, sink, saddle point, center, spiral source or spiral sink.

answer: sink

4. **Use Laplace transforms** to solve $y'' + 5y' + 4y = 14e^{3t}$, with $y(0) = 0, y'(0) = 0$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = -\frac{7}{6}e^{-t} + \frac{2}{3}e^{-4t} + \frac{1}{2}e^{3t}$

5. Find all equilibrium points of system below, and classify any two of them as a source, sink, saddle point, spiral source, spiral sink, or center.

$\frac{dx}{dt} = 2x(1 - \frac{x}{3}) - xy$
$\frac{dy}{dt} = -2y + 4xy$

answer: $(0,0)$ and $(3,0)$ are saddles, $(\frac{1}{2}, \frac{5}{3})$ is a spiral sink

6. Solve the differential equation of problem 4 **without** using Laplace transforms.

answer: $y(t) = -\frac{7}{6}e^{-t} + \frac{2}{3}e^{-4t} + \frac{1}{2}e^{3t}$