

Math 2326, Final Exam

Name _____

1. What does the solution of

$$y' = (y - 3)^2(5 - y)(2 + y), \text{ with } y(0) = -1$$

converge to, as $t \rightarrow \infty$?

answer: 3

2. Find the solution satisfying $x(0) = 2, y(0) = -2$, of

$$\begin{aligned} \frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= -4y \end{aligned}$$

answer: $x(t) = e^{2t} + e^{-4t}, y(t) = -2e^{-4t}$

3. Use Laplace transforms to solve $y'' + 10y' + 16y = 81e^t$, with $y(0) = 0, y'(0) = 0$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = \frac{-9}{2}e^{-2t} + \frac{3}{2}e^{-8t} + 3e^t$

4. Find the (only) equilibrium point of the system below with x and y both positive, and classify it as a source, sink, saddle point, spiral source, spiral sink, or center.

$$\begin{aligned}\frac{dx}{dt} &= x\left(2 - \frac{2x}{3} - y\right) \\ \frac{dy}{dt} &= y(-2 + 4x)\end{aligned}$$

answer: $(\frac{1}{2}, \frac{5}{3})$ is a spiral sink

5. A cup of coffee is initially $170^\circ F$ and is left in a room at where the air is $T_0 = 70^\circ F$. After 1 minute, the coffee has cooled to $150^\circ F$, what will its temperature be after 4 more minutes (5 minutes after it was 170°)? Assume Newton's law of cooling, that is, $\frac{dT}{dt} = -k(T(t) - T_0)$, where $T(t)$ is the coffee temperature, t is time and k is a constant.

answer: 102.77°

6. Find the general solution of $y'' - 6y' + 9y = 8\sin(t) - 6\cos(t)$

answer: $y(t) = c_1e^{3t} + c_2te^{3t} + \sin(t)$