

Math 2326, Final Exam

Name _____

1. For what range of initial values (A) will the solution of

$$y' = (y - 2)^3(5 - y)(3 + y), \text{ with } y(0) = A$$

converge (as $t \rightarrow \infty$) to $y = 2$? For what range will the solution converge to $y = 5$? (Hint: construct the phase line.)

answer: for $A = 2, y \rightarrow 2$, for $A > 2, y \rightarrow 5$

2. **Use Laplace transforms** to solve $y'' + 9y' + 18y = 210e^{4t}$, with $y(0) = 0, y'(0) = 0$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = -10e^{-3t} + 7e^{-6t} + 3e^{4t}$

3. Solve the differential equation of problem 2 **without** using Laplace transforms.

answer: $y(t) = -10e^{-3t} + 7e^{-6t} + 3e^{4t}$

4. Find both equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

$$\begin{aligned}\frac{dx}{dt} &= x^2 - y^2 \\ \frac{dy}{dt} &= xy + 25\end{aligned}$$

answer: $(5, -5)$ is a spiral source, $(-5, 5)$ is a spiral sink

5. a. Find the solution to the following system, with $x(0) = -3, y(0) = -2$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = e^{2t} \begin{bmatrix} 3 \\ 0 \end{bmatrix} + e^{3t} \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

- b. Find all equilibrium points of problem 5a, and classify each as a source, sink, saddle, spiral source, spiral sink or center.

answer: $(0, 0)$ is a source