

## Math 2326, Final Exam

Name \_\_\_\_\_

1. For what range of initial values ( $A$ ) will the solution of

$$y' = (y - 2)^2(y^2 - 25), \text{ with } y(0) = A$$

converge (as  $t \rightarrow \infty$ ) to  $y = 2$ ? For what range will the solution converge to  $y = -5$ ? (Hint: construct the phase line.)

answer: for  $2 \leq A < 5$ ,  $y \rightarrow 2$ , for  $A < 2$ ,  $y \rightarrow -5$

2. **Use Laplace transforms** to solve  $y'' - 4y = 24$ , with  $y(0) = 0$ ,  $y'(0) = 0$ . (Hints:  $L(y') = sL(y) - y(0)$  and  $L(e^{at}) = \frac{1}{s-a}$ ).

answer:  $y(t) = -6 + 3e^{2t} + 3e^{-2t}$

3. Solve the differential equation of problem 2 **without** using Laplace transforms.

answer:  $y(t) = -6 + 3e^{2t} + 3e^{-2t}$

4. Find both equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

$$\begin{aligned}\frac{dx}{dt} &= x^2 - y^2 \\ \frac{dy}{dt} &= xy - 36\end{aligned}$$

answer:  $(6, 6)$  is a spiral source,  $(-6, -6)$  is a spiral sink

5. a. Find the solution to the following system, with  $x(0) = 6, y(0) = 5$ .

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer:  $\begin{bmatrix} x \\ y \end{bmatrix} = e^{2t} \begin{bmatrix} 6 \\ 4 \end{bmatrix} + e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- b. Find all equilibrium points of problem 5a, and classify each as a source, sink, saddle, spiral source, spiral sink or center.

answer:  $(0, 0)$  is a saddle