

Math 2326, Final Exam

Name _____

1. a. Find the solution to the following system, with $x(0) = 2, y(0) = 8$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = e^{2t} \begin{bmatrix} 8 \\ 0 \end{bmatrix} + e^{-2t} \begin{bmatrix} -6 \\ 8 \end{bmatrix}$

- b. Classify the equilibrium point $(0, 0)$ as a sink, saddle, source, center, spiral sink or spiral source.

answer: *saddle*

- c. Now solve problem 1a as a partially decoupled system.

answer: $x = 8e^{2t} - 6e^{-2t}, y = 8e^{-2t}$

2. Solve $y'' + y = \delta_3(t)$, $y(0) = 0$, $y'(0) = 0$ using Laplace transforms.
(Hint: $L(\delta_a(t)) = e^{-as}$, $L(u_a(t)q(t-a)) = e^{-as}Q(s)$, $L(\sin(\omega t)) = \frac{\omega}{s^2 + \omega^2}$
and $L(y'') = s^2L(y) - sy(0) - y'(0)$.)

answer: $y(t) = u_3(t)\sin(t - 3)$

3. a. Find all three equilibrium points of the predator-prey system:

$$\begin{aligned}x' &= x\left(2 - \frac{2}{3}x - y\right) \\y' &= y(-2 + 4x)\end{aligned}$$

answer: $(0, 0)$, $(3, 0)$, $(\frac{1}{2}, \frac{5}{3})$

- b. Only one equilibrium point has both x and y greater than zero. Classify it as a sink, source, saddle, spiral sink, spiral source or center.

answer: $(\frac{1}{2}, \frac{5}{3})$ is spiral sink.

c. If $x(0) = 1, y(0) = 0$ what does x tend to, as $t \rightarrow \infty$?

answer: 3

d. If $x(0) = 0, y(0) = 1$ what does y tend to, as $t \rightarrow \infty$?

answer: 0

4. Find the general solution of $y'' + 6y' + 9y = 2\cos(3t)$.

answer: $y(t) = C_1e^{-3t} + C_2te^{-3t} + \frac{1}{9}\sin(3t)$

5. A wine chilled at $10^\circ C$ is suddenly moved to a room where the temperature is $30^\circ C$. If at $t = 0$ the wine is warming up at the rate of $5^\circ C/hour$, when will the wine reach $15^\circ C$? Assume Newton's law of cooling, that is, $\frac{dT}{dt} = -k(T - T_0)$, where T is temperature, t is time, k is a constant, and T_0 is the room temperature.

answer: $t = 1.150$ hours