Math 2326, Final Exam

Name _____

1. For what range of initial values (a) will the solution of

   $y' = (y-2)^2(8-y)(1+y)^3$, with $y(0) = a$

   converge (as $t \to \infty$) to $y = 2$? (Hint: construct the phase line.)

   answer: for $-1 < a \leq 2$

2. If the eigenvalues (λ_i) and eigenvectors (z_i) of the 3 by 3 matrix A are:

   $\lambda_1 = 4, z_1 = (1, 3, -2); \lambda_2 = -3, z_2 = (2, 0, 1); \lambda_3 = 9, z_3 = (0, 0, 1)$

   a. Write the general solution of $y' = Ay$, where $y = (y_1, y_2, y_3)$.

   answer: $y = C_1 e^{4t} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} + C_2 e^{-3t} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + C_3 e^{9t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

b. Write the solution that satisfies $y_1(0) = 0, y_2(0) = 0, y_3(0) = 5$.

answer: $y = 5e^{9t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3. **Use Laplace transforms** to solve $y' + y = e^{-2t}$, with $y(0) = 2$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = 3e^{-t} - e^{-2t}$

4. Find the general solution of $y'' + 6y' + 9y = 2cos(3t)$.

answer: $y(t) = C_1e^{-3t} + C_2te^{-3t} + \frac{1}{9}sin(3t)$

5. Find both equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

$\frac{dx}{dt} = x^2 - y^2$
$\frac{dy}{dt} = xy - 25$

answer: $(5, 5)$ is spiral source, $(-5, -5)$ is spiral sink

6. The following MATLAB program is to use Euler's method to solve the system of differential equations of problem 5, with initial conditions $x(1) = 2, y(1) = 3$. Finish the seven incomplete statements. (You don't need to use correct MATLAB syntax).

```
-->  t = 1;
-->  x = 2;
-->  y = 3;
     h = 0.001;
     for i=1:1000
-->     f1 = x^2-y^2;
-->     f2 = x*y-25;
-->     x = x + h*f1;
-->     y = y + h*f2;
        t = t + h;
     end
```