Math 2326, Final Exam

Name ______________________________

Work any 5 problems. Indicate clearly which problem NOT to be graded.

1. For what range of initial values (A) will the solution of

   $y' = (y-2)^3(3-y)sin(\pi y/5)$, with $y(0) = A$

   converge (as $t \to \infty$) to $y = 3$? (Hint: construct the phase line.)

   answer: $2 < A < 5$

2. If the eigenvalues (λ_i) and eigenvectors (z_i) of the 3 by 3 matrix A are:

   $\lambda_1 = 4, z_1 = (1, 3, -2); \lambda_2 = -3, z_2 = (2, 0, 1); \lambda_3 = 7, z_3 = (0, 0, 1)$

   find the solution of $y' = Ay,\ y_1(0) = 1, y_2(0) = 3, y_3(0) = 4,$ where $y = (y_1, y_2, y_3)$.

   answer: $y = e^{4t}\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} + 6e^{7t}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

3. Find all equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

$\frac{dx}{dt} = x^3 - y^3$
$\frac{dy}{dt} = xy - 4$

answer: $(2, 2)$ is source, $(-2, -2)$ is saddle

4. **Use Laplace transforms** to solve $y' - y = 8e^{-3t}$, with $y(0) = 2$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = 4e^t - 2e^{-3t}$

5. Solve $y'' + 6y' + 10y = 74e^{3t}$, with $y(0) = 4, y'(0) = 1$.

answer: $y(t) = e^{-3t}sin(t) + 2e^{-3t}cos(t) + 2e^{3t}$

6. Take one step of Euler's method to solve $y' = (1+2t)y^2, y(0) = 1$, with $h = 0.01$. Then solve the equation analytically and evaluate the exact solution at $t = 0.01$, to compare with the Euler approximation.

answer: Euler: $y(0.01) = 1.01$, exact solution: $y(t) = 1/(1 - t - t^2)$, so $y(0.01) = 1.0102$.