Math 2326, Final Exam

Name ______________________________

1. Reduce the second order equation $y''(t) - 7y'(t) + 10y(t) = 0, y(0) = 0, y'(0) = 6$ to a system of first order equations ($x' = Ax, x(0) = x_0$, where $x(t)$ is a vector, A is a 2 by 2 matrix).

answer:

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with $x_1(0) = 0, x_2(0) = 6$

2. Solve the system $x' = Ax, x(0) = x_0$ of problem 1, using the techniques of chapter 3, that is, by finding the eigenvalues and eigenvectors of A, and writing the general solution for $x(t)$, then finding the values for the arbitrary constants which make $x(0) = x_0$.

answer: $x(t) = e^{2t} \begin{bmatrix} -2 \\ -4 \end{bmatrix} + e^{5t} \begin{bmatrix} 2 \\ 10 \end{bmatrix}$

3. Now solve the second order equation of problem 1 (with initial conditions) using Laplace transforms. (Hint: $L(e^{at}) = \frac{1}{s-a}$, and $L(y') = sL(y) - y(0)$.)

answer: $y(t) = -2e^{2t} + 2e^{5t}$

4. Now solve the second order equation of problem 1 (with initial conditions) by trying solutions of the form $y(t) = e^{rt}$ and determining the values of r (the roots of the characteristic polynomial) that make this a solution.

answer: $y(t) = -2e^{2t} + 2e^{5t}$

5. a. Find all three equilibrium points of the preditor-prey system:

$$\begin{aligned} x' &= x(2 - 2x - y) \\ y' &= y(-2 + 4x) \end{aligned}$$

answer: $(0,0), (1,0), (\frac{1}{2}, 1)$

b. If $x(0) = 3, y(0) = 0$ what does $x(t)$ tend to as $t \to \infty$?

answer: When $y = 0$, $x \to 1$.

c. Only one equilibrium point has both x and y greater than zero. Classify it as a sink, source, saddle, spiral sink, spiral source or center.

answer: $(\frac{1}{2}, 1)$ is spiral sink.