Math 2326, Final Exam

Name ______________________________

1. For what range of initial values (A) will the solution of

   $y' = (y-2)^3(8-y)(1+y)$, with $y(0) = A$

   converge (as $t \to \infty$) to $y = 8$? (Hint: construct the phase line.)

   answer: for $A > 2, y \to 8$

2. If the eigenvalues (λ_i) and eigenvectors (z_i) of the 3 by 3 matrix A are:

   $\lambda_1 = 4, z_1 = (1, 3, -2); \lambda_2 = -3, z_2 = (2, 0, 1); \lambda_3 = 7, z_3 = (0, 1, 0)$

   find the solution of $y' = Ay$, where $y = (y_1, y_2, y_3)$, which satisfies $y(0) = (0, 1, 1)$.

   answer: $y = -0.4e^{4t}\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} + 0.2e^{-3t}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + 2.2e^{7t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

3. Solve $y' = e^{y+t}$, with $y(0) = ln(\frac{1}{2})$.

answer: $y(t) = -ln(3 - e^t)$

4. **Use Laplace transforms** to solve $y'' + 9y' + 18y = 54e^{3t}$, with $y(0) = 0, y'(0) = 0$. (Hints: $L(y') = sL(y) - y(0)$ and $L(e^{at}) = \frac{1}{s-a}$).

answer: $y(t) = -3e^{-3t} + 2e^{-6t} + e^{3t}$

5. Verify that $(30, 40)$ is an equilibrium point of the system below, and classify it as a source, sink, saddle point, spiral source, spiral sink, or center.

$$\frac{dx}{dt} = -8x^2 - 6xy + 480x$$
$$\frac{dy}{dt} = -x^2y - y^3 + 2500y$$

answer: eigenvalues of Jacobian are $-101, -3339$, so $(30, 40)$ is a sink

6. Find the general solution of $y'' + 6y' + 9y = 2cos(3t)$.

answer: $y(t) = C_1e^{-3t} + C_2te^{-3t} + \frac{1}{9}sin(3t)$