Math 2326, Final Exam

Name ______________________________

1. Reduce the second order equation $y''(t) - 6y'(t) + 5y(t) = 0, y(0) = 3, y'(0) = 7$ to a system of first order equations ($x' = Ax, x(0) = x_0$, where $x(t)$ is a vector, A is a 2 by 2 matrix).

answer:

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

with $x_1(0) = 3, x_2(0) = 7$

2. Solve the system $x' = Ax, x(0) = x_0$ of problem 1, using the techniques of chapter 3, that is, by finding the eigenvalues and eigenvectors of A, and writing the general solution for $x(t)$, then finding the values for the arbitrary constants which make $x(0) = x_0$.

answer: $x(t) = e^{5t} \begin{bmatrix} 1 \\ 5 \end{bmatrix} + e^{t} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

3. Now solve the second order equation of problem 1 (with initial conditions) using Laplace transforms. (Hint: $L(e^{at}) = \frac{1}{s-a}$, and $L(y') = sL(y) - y(0)$.)

answer: $L(y) = \frac{3s-11}{s^2-6s+5}; y(t) = e^{5t} + 2e^t$

4. Now solve the second order equation of problem 1 (with initial conditions) by trying solutions of the form $y(t) = e^{rt}$ and determining the values of r that make this a solution.

answer: $y(t) = e^{5t} + 2e^t$

5. Solve $y' = e^{y+t}$, with $y(0) = 0$.

answer: $y(t) = -ln(2 - e^t)$

6. Find the general solution of $y'' + 4y' + 4y = 2cos(4t)$.

answer: $y(t) = C_1 e^{-2t} + C_2 te^{-2t} + 0.08\ sin(4t) - 0.06\ cos(4t)$

7. Find both equilibrium points of the system below, and classify each as a source, sink, saddle point, spiral source, spiral sink, or center.

$\frac{dx}{dt} = x^3 - y^3$
$\frac{dy}{dt} = xy - 1$

answer: $(1, 1)$ is spiral source, $(-1, -1)$ is saddle