Math 3323, Test I

Name ____________________________

1. If

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 3 & 5 \\ 2 & 4 & 6 & 1 \\ -1 & -2 & -3 & 7 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3, x_4) = (\frac{1}{3}, 0, 0, \frac{1}{3}) + x_2(-2, 1, 0, 0) + x_3(-3, 0, 1, 0)$.

2. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix which has r nonzero rows after reduction to row echelon form. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible (more than one of these options is usually possible). If you write "many", put in parenthesis the number of arbitrary coefficients in the general solution. (If you can't give a single value for this number, indicate the range of possible values.)

   a. m=3, n=7, r=2 and b=0. (answer: many(5))
   b. m=7, n=3, r=3. (answer: no, unique)
   c. m=7, n=3, r=3 and b=0. (answer: unique)
   d. m=6, n=5, r=4. (answer: no, many(1))
   e. m=4, n=5, b=0 and the rows of A are independent. (answer: many(1))
   f. m=n, and the columns of A are independent. (answer: unique)
   g. m=n, and the columns of A are dependent. (answer: no, many(1 or more))
   h. m=n, b=0, and A is singular. (answer: many(1 or more))

3. Find the polynomial of degree 2 or less, $a_2x^2 + a_1x + a_0$ which passes through the points $(-1, -6), (0, 3), (1, 12)$. Solve the linear system by hand, and show your work.

answer: $9x + 3$

4 Find the inverse of

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 3 & 5 \\ 0 & 2 & 7 \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} 11 & 1 & -7 \\ -7 & 0 & 4 \\ 2 & 0 & -1 \end{bmatrix},$$

5. If

$$A^{-1} = \begin{bmatrix} 0 & -1 & 2 \\ 3 & 1 & 3 \\ 2 & 0 & 7 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3 & 3 & 2 \\ 3 & 0 & 4 \\ -1 & 2 & 3 \end{bmatrix},$$

find $(AB)^{-1}$.

answer:

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 13 & 0 & 29 \\ 8 & -3 & 34 \\ 12 & 3 & 25 \end{bmatrix},$$