Math 3323, Test I

Name ____________________________

1. If

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 2 & 1 & 3 & -1 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3, x_4, x_5) = (1, -2, 0, 0, 0) + x_3(-1, -1, 1, 0, 0) + x_4(-1, 3, 0, 1, 0) + x_5(2, -5, 0, 0, 1)$.

2. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix which has r nonzero rows after reduction to row echelon form. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible (more than one of these options may be possible).

   a. m=3, n=5, r=2 and b=0. (answer: many)

   b. m=7, n=3, r=3. (answer: no, unique)

   c. m=n, b=0, and A is singular. (answer: many)

3. Find constants a, b, c such that $y(x) = a + be^x + ce^{-x}$ satisfies $y(0) = 3, y'(0) = 0, y''(0) = 2$.

answer: $a = b = c = 1$

4. Find a such that these vectors are linearly dependent:

$$u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, w = \begin{bmatrix} a \\ 1 \\ 4 \end{bmatrix}$$

answer: $a = -3$

5. Find the inverse of

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \\ 2 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix},$$

6. Show that $x^T Q^T Q x \geq 0$ for any matrix Q and vector x, assuming the product Qx exists.

answer: $x^T Q^T Q x = (Qx)^T Q x = \|Qx\|^2 \geq 0$