Math 3323, Test I

Name `_____________________________`

1. If

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3, x_4, x_5) = (1, 2, 0, 1, 0) + x_3(1, -2, 1, 0, 0) + x_5(1, -1, 0, -1, 1)$

2. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix which has r nonzero rows after reduction to row echelon form. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible (more than one of these options may be possible).

   a. m=3, n=7, r=2 and b=0. (many)
   b. m=7, n=3, r=3. (no,unique)
   c. m=7, n=3, r=3 and b=0. (unique)
   d. m=n, and the columns of A are independent. (unique)
   e. m=n, and the rows of A are dependent. (no,many)

3. Find the polynomial of degree 3 or less, $p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ which satisfies $p(0) = 2, p'(0) = 3, p(1) = 8, p'(1) = 10$.

answer: $p(x) = x^3 + 2x^2 + 3x + 2$

4. Find the inverse of

$$A = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix},$$

5. Show that the vectors below are linearly dependent, by finding x_1, x_2, x_3 not all zero, such that $x_1 u + x_2 v + x_3 w = 0$:

$$u = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 5 \\ 2 \\ 9 \end{bmatrix}$$

answer: $x_1 = 1, x_2 = -3, x_3 = 1$ (or any multiple of these)

6. If $A = I - avv^T$, where v is a nonzero vector and $a = 2/(v^T v)$, show that $A^T = A^{-1}$ (A is orthogonal).

answer: $A^T A = (I - avv^T)^T (I - avv^T) = (I - avv^T)(I - avv^T) = I - 2avv^T + a^2 v(v^T v)v^T = I - 2avv^T + 2avv^T = I.$