Math 3323, Test I

Name ____________________________

1. If

$$A = \begin{bmatrix} 2 & 2 & -1 \\ -2 & -2 & 4 \\ 2 & 2 & 5 \\ -2 & -2 & -2 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 5 \\ -3 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3) = (\frac{5}{6}, 0, \frac{2}{3}) + x_2(-1, 1, 0)$.

2. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix which has r nonzero rows after reduction to row echelon form. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible (more than one of these options may be possible).

   a. m=3, n=7, r=2 and b=0. (answer: many)
   b. m=7, n=3, r=3. (answer: no, unique)
   c. m=7, n=3, r=3 and b=0. (answer: unique)
   d. m=6, n=5, r=4. (answer: no, many)
   e. m=4, n=5, b=0 and the rows of A are independent. (answer: many)
   f. m=n, and the columns of A are independent. (answer: unique)
   g. m=n, and the columns of A are dependent. (answer: no, many)
   h. m=n, b=0, and A is singular. (answer: many)

3. Find the polynomial of degree 2 or less, $a_2x^2 + a_1x + a_0$ which passes through the points $(-1, -6), (0, 3), (1, 12)$. Solve the linear system by hand, and show your work.

answer: $9x + 3$

4 Find the inverse of

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 3 & 5 \\ 0 & 2 & 7 \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} 11 & 1 & -7 \\ -7 & 0 & 4 \\ 2 & 0 & -1 \end{bmatrix},$$

5. Find a such that these vectors are linearly dependent:

$$u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, w = \begin{bmatrix} -3 \\ 1 \\ a \end{bmatrix}$$

answer: $a = 4$