Math 3323, Test I

Name ____________________________

1. If

$$A = \begin{bmatrix} 4 & 0 & 12 & 8 & -8 \\ 6 & 0 & 2 & 12 & -12 \\ 2 & 0 & 2 & 4 & -4 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

find the general solution of $Ax = b$, and write in vector form.

answer: $(x_1, x_2, x_3, x_4, x_5) = x_2(0, 1, 0, 0, 0) + x_4(-2, 0, 0, 1, 0) + x_5(2, 0, 0, 0, 1)$.

2. Suppose a linear system $Ax = b$ is solved, where A is an m by n matrix. For each case described below, write "no", "unique" and/or "many" to indicate that no solution, a unique solution, or many solutions are possible (more than one of these options may be possible).

   a. m=4, n=5 and b=0. (answer: many)
   b. m=4, n=3. (answer: no, unique, many)
   c. m=4, n=3 and b=0. (answer: unique, many)
   d. m=3, n=3, and there are 3 nonzero rows after reduction to echelon form. (answer: unique)
   e. m=n, b=0, and the columns of A are dependent. (answer: many)
   f. m=n and A is singular. (answer: no, many)

3. Find the polynomial of degree 3 or less, $p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ which satisfies $p(0) = -1, p'(0) = 1, p(1) = 2, p'(1) = 6$.

answer: $p(x) = x^3 + x^2 + x - 1$

4. Find a such that these vectors are linearly dependent:

$$u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, w = \begin{bmatrix} -3 \\ 1 \\ a \end{bmatrix}$$

answer: $a = 11$

5. If

$$A^{-1} = \begin{bmatrix} 3 & -1 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

find $(AB)^{-1}$.

answer:

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 14 & -8 & 0 \\ 0 & 0 & 5 \end{bmatrix},$$