

Math 3323, Test II

Name _____

1. What is the volume of a parallelopiped with edges

$$u = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

answer: 24

2. If

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 1 & 5 & 4 \\ 1 & 2 & 4 & -1 \end{bmatrix},$$

- a. Find a basis for the subspace spanned by the rows of A.

answer: $\langle 1, 0, 2, 3 \rangle$ and $\langle 0, 1, 1, -2 \rangle$ is one basis.

- b. Find a basis for the null space of A.

answer: $\langle -2, -1, 1, 0 \rangle$ and $\langle -3, 2, 0, 1 \rangle$ is one basis.

3. If $[u_1, \dots, u_p]$ is an orthogonal set of nonzero vectors,
- Prove that this set is linearly independent.

- If $w = c_1u_1 + \dots + c_pu_p$, find c_i .

answer: $c_i = w^T u_i / u_i^T u_i$

4. Find the least-squares linear fit to the points $(-1, -1), (0, 1), (1, 2), (2, 4)$.

answer: $y = 1.6t + 0.7$

5. True or False (remember that a statement is false unless it is *always* true):
- a. The cross product is only defined for vectors in R^3 . (true)
 - b. The line $x = 4+2t, y = 7+2t, z = 5+4t$ and the plane $x+y-z = 2$ do not intersect. (true)
 - c. $x = 4+2t, y = 7+2t, z = 5+4t$ and $x = 2+t, y = 5+t, z = 1+2t$ are the same line. (true)
 - d. A set of 3 vectors in a 5-dimensional subspace may be a spanning set for the subspace. (false)
 - e. A set of 3 vectors in a 5-dimensional subspace may be linearly dependent. (true)
 - f. A set of 7 vectors in a 5-dimensional subspace may be linearly independent. (false)
 - g. The dimensions of the subspaces spanned by the rows and columns of an $m \times n$ matrix are the same. (true)
 - h. The range of AB is a subset of the range of A . (true)
 - i. The intersection of two subspaces is a subspace. (true)
 - j. The null space of the $n \times n$ zero matrix is R^n . (true)
 - k. The set of solutions to $Ax = b$ is a subspace. (false)
 - l. $u \times v = \|u\| \|v\| \sin(\theta)$, where θ is the angle between u and v . (false)