

Math 3323, Test II

Name _____

1. Which of the following subsets are subspaces? For those that are subspaces, give the dimension.

- a. The range of matrix

$$A = \begin{bmatrix} 4 & 0 & 4 & 8 & -8 \\ 6 & 0 & 6 & 12 & -12 \\ 2 & 0 & 2 & 4 & -4 \end{bmatrix}$$

answer: yes, dimension=1

- b. The set of solutions to $Ax = b$ where A is the matrix in part a and

$$b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

answer: no

- c. The set of solutions to $Ax = 0$ where A is the matrix in part a.

answer: yes, dimension=4

- d. The set of vectors in R^n which are perpendicular to a given nonzero vector.

answer: yes, dimension=n-1

- e. The line in R^3 , $x = -1 + t$, $y = 2t$, $z = 3 - t$.

answer: no

- f. The plane in R^3 , $x + y + z = 2$.

answer: no

2. Find a basis for the subspace spanned by

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} -6 \\ -12 \\ 6 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 10 \\ 12 \\ 6 \end{bmatrix}, v_5 = \begin{bmatrix} 7 \\ 8 \\ 5 \end{bmatrix}$$

answer: $(1, 2, -1), (0, 1, -2)$ is one basis.

3. If

$$u = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

- a. Find $u \times v$.
answer: $(1, -2, 1)$
 - b. Find the volume of a parallelepiped with edges u, v, w .
answer: 4
 - c. Find the equation of the plane through the origin parallel to the vectors u and v .
answer: $x - 2y + z = 0$
 - d. Find the equations of a line through $(2, 3, 2)$ perpendicular to the plane in part c.
answer: $x = 2 + t, y = 3 - 2t, z = 2 + t$
- 4.
- a. What are the range and null space of the n by n identity matrix?
answer: Range is all of R^n , null space is origin.
 - b. What are the range and null space of the n by n zero matrix?
answer: Range is origin, null space is all of R^n
 - c. True or False: The null space of B is a subset of the null space of AB .
answer: True
 - d. True or False: The range of AB is a subset of the range of A .
answer: True