

## Math 3323, Test II

Name \_\_\_\_\_

1. Which of the following subsets are subspaces? For those that are subspaces, give the dimension.

- a. The null space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 0 \\ 2 & 1 & 5 & 4 & 0 \\ 1 & 2 & 4 & -1 & 0 \end{bmatrix}$$

answer: yes, dimension=3

- b. The row space of the matrix in part a.

answer: yes, dimension=2

- c. The range of the matrix in part a.

answer: yes, dimension=2

- d. The line in  $R^3$ ,  $x = 1 + t$ ,  $y = 2t$ ,  $z = -t$ .

answer: no

- e. The plane  $z = 0$  in  $R^3$ .

answer: yes, dimension=2

- f. The union of the planes  $x = 0$  and  $z = 0$  in  $R^3$ .

answer: no

2. a. Find a basis for the range of the matrix

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 0 & 6 \\ 1 & 0 & 2 \end{bmatrix}$$

answer:  $\langle 2, 3, 1 \rangle$

- b. Find a basis for the null space of this matrix.

answer:  $\langle 0, 1, 0 \rangle$  and  $\langle 2, 0, -1 \rangle$

3. If

$$u = \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix},$$

- a. Find  $u \times v$ .

answer:  $\langle 8, 4, 2 \rangle$

- b. Find the equation of the plane through  $(2, 1, 2)$  parallel to the vectors  $u$  and  $v$ .

answer:  $8x + 4y + 2z = 24$

- c. Find the equations of a line through  $(2, 1, 2)$  perpendicular to the plane in part b.

answer:  $x = 2 + 8t, y = 1 + 4t, z = 2 + 2t$

- d. Find the area of a triangle with edges  $u$  and  $v$ .

answer:  $\sqrt{21}$

4. True or False:

- a. The dot product is only defined for vectors in  $R^3$ . (false)
- b. The line  $x = 4+2t, y = 7+2t, z = 5-4t$  and the plane  $x+y+z = 2$  intersect. (false)
- c. Any set of 5 independent vectors in a 5-dimensional subspace is a basis for the subspace. (true)
- d. A set of 7 vectors in a 5-dimensional subspace may be linearly independent. (false)
- e. A set of 4 vectors in a 5-dimensional subspace may be linearly dependent. (true)
- f. A set of 4 vectors in a 5-dimensional subspace may be a spanning set for the subspace. (false)
- g. The dimensions of the subspaces spanned by the rows and columns of an  $m \times n$  matrix are the same. (true)
- h. The intersection of two subspaces is a subspace. (true)
- i. The null space of the  $m$  by  $n$  zero matrix is  $R^n$ . (true)
- j. The null space of  $B$  is a subset of the null space of  $AB$ . (true)
- k. The range of  $AB$  is a subset of the range of  $A$ . (true)
- l. The lines  $x = 1 - t, y = 2 - 2t, z = 3 - 3t$  and  $x = t, y = 2t, z = 3t$  are the same line. (true)