Math 3323, Final

Name ______________________________

1. If A =

$$\begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

   a. Find all eigenvalues of A.

   answer: $\lambda = 2, 3$

   b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

   answer: for $\lambda = 2$, basis is $[(1,0,0),(0,-2,1)]$, for $\lambda = 3$, basis is $[(1,1,0)]$.

2. Find the eigenvalues and corresponding eigenvectors for A =

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

answer: $\lambda_1 = 1 + 2i$, $x_1 = (i, 1)$ or $(1, -i)$; $\lambda_2 = 1 - 2i$, $x_2 = (-i, 1)$ or $(1, i)$.

3. Find the determinant of A =

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 0 \\ 4 & 4 & 1 & 2 \\ 0 & 3 & 1 & 4 \end{bmatrix}$$

answer: 18

4. Find the general solution of:

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = 6x - y$$

answer: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 e^{-3t} \begin{bmatrix} 1 \\ -3 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

5. Find a basis for the subspace spanned by the four vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

answer: $[(1, 2, 1), (0, 1, -2)]$ (other answers possible)

6. Find the inverse of

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix},$$

answer:

$$A^{-1} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1.5 & -2 \\ 0 & 0 & -0.5 & 1 \end{bmatrix},$$

7. Define:

a. An orthogonal matrix
answer: $A^T A = I$

b. A symmetric matrix
answer: $A^T = A$

c. A positive definite matrix
answer: symmetric, with all eigenvalues positive

d. A singular matrix
answer: $\det(\mathrm{A}) = 0$

8. Calculate A^{10}, if A =

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Hint: $S^{-1}AS = D$, where

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, S = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

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answer: $A^{10} = SD^{10}S^{-1} =$

$$\begin{bmatrix} 29525 & -29524 \\ -29524 & 29525 \end{bmatrix}$$

9. Write the equations for:

   a. The line through the points (1,-1,2) and (3,3,3).

      answer: $x = 1 + 2t, y = -1 + 4t, z = 2 + t$

   b. The plane through (1,-1,2), perpendicular to this line.

      answer: $2(x-1) + 4(y+1) + (z-2) = 0$