Math 3323, Final

Name ______________________________

1. If A =

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

   a. Find all eigenvalues of A.

   answer: $\lambda = 2$

   b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

   answer: for $\lambda = 2$, basis is $[(1,0,0),(0,0,1)]$.

2. Find the eigenvalues and corresponding eigenvectors for A =

$$\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

answer: $\lambda_1 = 2+i$, $x_1 = (1, 1+i)$ or $(1-i, 2)$; $\lambda_2 = 2-i$, $x_2 = (1, 1-i)$ or $(1+i, 2)$.

3. Find the general solution of:

$$\frac{dx}{dt} = x + y + z$$
$$\frac{dy}{dt} = y + z$$
$$\frac{dz}{dt} = 3y + 3z$$

answer: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_2 e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_3 e^{4t} \begin{bmatrix} 4 \\ 3 \\ 9 \end{bmatrix}$

4. Write the equations for:

   a. The plane through (0,0,0),(1,-1,0) and (0,1,1).

   answer: $-x - y + z = 0$

   b. The line through the (1,2,3) perpendicular to this plane.

   answer: $x = 1 - t, y = 2 - t, z = 3 + t$

c. Does the plane of part (a) and/or the line of part (b) represent a subspace of R^3?

answer: The plane is a subspace.

5. If A =

$$\begin{bmatrix} 1 & -6 & 0 & 10 & 7 \\ 2 & -12 & 0 & 12 & 8 \\ -1 & 6 & 0 & 6 & 5 \end{bmatrix}$$

a. Find a basis for the null space of A.

answer: One basis is $[(6,1,0,0,0),(0,0,1,0,0),(0.5,0,0,-0.75,1)]$.

b. Find a basis for the range of A.

answer: One basis is $[(1,2,-1),(0,1,-2)]$.

c. What is the rank of A? What is the nullity? (Hint: they must add up to the number of columns).

answer: rank=2, nullity=3.

6. If $[u_1, ..., u_p]$ is an orthogonal set of nonzero vectors,

   a. Prove that this set is linearly independent.

   b. If $w = c_1 u_1 + ... + c_p u_p$, find c_i.

   answer: $c_i = w^T u_i / u_i^T u_i$