Math 3323, Final

Name ______________________________

1. If A =

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

a. Find all eigenvalues of A.

answer: $\lambda = 0, 2$

b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

answer: for $\lambda = 0$, basis is $[(0, 1, -1)]$, for $\lambda = 2$, basis is $[(1, 0, 0), (0, 1, 1)]$.

c. Is A defective (explain answer)?

answer: no, there are 3 independent eigenvectors.

2. Find the eigenvalues and corresponding eigenvectors for A =

$$\begin{bmatrix} 2 & 2 \\ -4 & 6 \end{bmatrix}$$

answer: $\lambda_1 = 4 + 2i$, $x_1 = (1, 1 + i)$; $\lambda_2 = 4 - 2i$, $x_2 = (1, 1 - i)$.

3. a. If $det(A) = 10, det(B^{-1}) = 2$, what is $det(B^2AB^{-1})$? answer: 5.

   b. Show that if A and B are orthogonal, AB is orthogonal. answer: $(AB)^T(AB) = B^TA^TAB = B^TB = I$

   c. What can you say about the eigenvalues of a symmetric matrix? answer: they are real.

   d. What can you say about the eigenvalues of a positive definite matrix? answer: they are real and positive.

   e. What can you say about the eigenvalues of an orthogonal matrix? answer: they have absolute value 1.

   f. What can you say about the eigenvalues of A and SAS^{-1}? answer: they are the same

4. Find the determinant of A =

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

answer: 80

5. Find the general solution of (hint: see problem 1):

$$\frac{dx}{dt} = 2x$$

$$\frac{dy}{dt} = y + z$$

$$\frac{dz}{dt} = y + z$$

answer: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_3 e^{2t} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

6. Find the inverse of the matrix of problem 4.

answer:

$$\begin{bmatrix} 0 & 0 & 0.25 & 0 \\ 1 & 0 & -0.25 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$$

7. If A =

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 7 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$$

a. Find a basis for the subspace spanned by the rows of A.

answer: $[(1, 2, 3, 1), (0, 1, 1, -1)]$ (other answers possible)

b. Find a basis for the subspace spanned by the columns of A.

answer: $[(1, 2, 1), (0, 1, -2)]$ (other answers possible)

c. What is the rank of A? answer: 2

8. Write the equations for:

   a. The plane through the points (0,0,0),(1,-1,2), and (3,3,3).

      answer: $-9x + 3y + 6z = 0$ or $-3x + y + 2z = 0$.

   b. The line through (1,-1,2) perpendicular to this plane.

      answer: $x = 1 - 9t, y = -1 + 3t, z = 2 + 6t$