Math 3323, Final

Name ______________________________

1. If A =

$$\begin{bmatrix} 3 & -1 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

a. Find all eigenvalues of A.

answer: $\lambda = 1, 5$

b. For each eigenvalue, find a basis for the subspace of eigenvectors (the eigenspace).

answer: for $\lambda = 1$, basis is $[(1, 2, 0)]$, for $\lambda = 5$, basis is $[(1, -2, 0), (0, 0, 1)]$.

c. Find matrices S and D, where D is diagonal, such that $A = SDS^{-1}$.

answer: $S = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

2. Find the eigenvalues of A =

$$\begin{bmatrix} 3 & -5 & 0 & 0 \\ 5 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

answer: $3+5i, 3-5i, 1-\sqrt{6}, 1+\sqrt{6}$.

3. Find the general solution of (hint: see problem 1):

$$\begin{aligned} \frac{dx}{dt} &= 3x - y \\ \frac{dy}{dt} &= -4x + 3y \\ \frac{dz}{dt} &= 5z \end{aligned}$$

answer: $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 e^t \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + C_3 e^{5t} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

4. If A =

$$\begin{bmatrix} 4 & 0 & 12 & 8 & -8 \\ 6 & 0 & 2 & 12 & -12 \\ 2 & 0 & 2 & 4 & -4 \end{bmatrix}$$

a. Find a basis for the range of A.

answer: $[(2,3,1),(0,4,1)]$ (other answers possible)

b. Find a basis for the null space of A.

answer: $[(0,1,0,0,0),(-2,0,0,1,0),(2,0,0,0,1)]$ (other answers possible)

c. What is the rank and what is the nullity of A? What do the rank and nullity always add up to?

answer: rank=2, nullity=3. rank+nullity=n (number of columns)

5. Write the equations for:

a. The plane through the points (-1,-1,-1),(1,1,2), and (1,3,3).

answer: $-4(x+1)-2(y+1)+4(z+1)=0$ or $2x+y-2z+1=0$.

b. The line of intersection of the planes $x + z = 4, 2x - y + 3z = 3$.

answer: $x = 4 - t, y = 5 + t, z = t$ (other answers possible)

6. Find the least squares linear fit $y = b+mx$ to the data points $(-2, 2), (0, 1), (1, 0), (2, 0)$.

answer: $y = (31 - 19x)/35$.

7. Show that an orthogonal set of three nonzero vectors $[u, v, w]$ is linearly independent.

answer: $au + bv + cw = 0$ implies $u \bullet (au + bv + cw) = 0, a\|u\|^2 = 0$ so $a = 0$, etc.