Math 4329, Test I

Name _____

1. a. If $f(x) = x^4 - 10x^3 + x + 3$, find the Taylor polynomial $T_3(x)$ of degree 3 which matches f, f', f'' and f''' at $a = 1$.

   b. Use the Taylor remainder formula to get a reasonable bound (in terms of x) on the error $|f(x) - T_3(x)|$ at x.

2. A certain computer stores floating point numbers in a 128-bit word, which includes 1 sign bit, 13 bits for the exponent, and 114 bits for the mantissa (significand). Assuming a normalized binary form is used $(1.xxxxx..._2 * 2^e)$ **approximately** what are:

   a. the underflow limit (smallest positive number)

   b. the overflow limit (largest positive number)

   c. the machine precision (smallest $\epsilon > 0$ such that $1 + \epsilon > 1$)

3. Compute the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $10^{-5}, 10^{-7}$ and 10^{-12}.

4. The polynomial $x^3 - x^2 - x - 1$ has one real root, at $x = 1.839$. We can write $x^3 - x^2 - x - 1 = 0$ in the form $x^3 = x^2 + x + 1$, or $x = 1 + \frac{1}{x} + \frac{1}{x^2}$ and try the iteration $x_{n+1} = 1 + \frac{1}{x_n} + \frac{1}{x_n^2}$.

   Will this converge, for x_0 near 1.839? Justify your answer **theoretically**.

5. Show how Newton's method could be used to find $b^{\frac{n}{m}}$ for $b > 0$, where n and m are positive integers, without doing anything other than add, subtract, multiply and divide. Hint: first write a function $f(x)$ whose root is $b^{\frac{1}{m}}$. Remember that an **integer** power of b can be computed using multiplication.

6. For the secant method, $e_{n+1} \approx M e_n e_{n-1}$. If the order of the secant method is α (ie, $e_{n+1} \approx C e_n^\alpha$), show that α must satisfy $\alpha = 1 + 1/\alpha$.