

## Math 4329, Test I

Name \_\_\_\_\_

1.
  - a. If  $f(x) = x^4 + 2x$ , find the Taylor polynomial  $T_3(x)$  of degree 3 which matches  $f, f', f''$  and  $f'''$  at  $a = 1$ .
  
  
  
  
  
  
  
  
  
  
  - b. Use the Taylor remainder formula to get a reasonable bound on the error  $|f(x) - T_3(x)|$  in the interval  $-1 \leq x \leq 3$ .
  
  
  
  
  
  
  
  
  
  
2. A certain computer stores floating point numbers in a 96-bit word, which includes 1 sign bit, 15 bits for the exponent, and 80 bits for the mantissa (significand). Assuming a normalized binary form is used ( $1.xxxxx\dots_2 * 2^e$ ), and  $e + 16383$  is stored in the 15-bit exponent part of the number, **approximately** what are:
  - a. the underflow limit (smallest positive number)
  
  
  
  
  
  
  
  
  
  
  - b. the overflow limit (largest positive number)
  
  
  
  
  
  
  
  
  
  
  - c. the machine precision (smallest  $\epsilon > 0$  such that  $1 + \epsilon > 1$ )

3. Write the quadratic formula root  $[-b + \sqrt{b^2 - 4ac}]/(2a)$  in a form so that there are no serious problems with roundoff error, when  $b$  is positive and very large compared to  $ac$ .
  
  
  
  
  
  
  
  
  
  
4. If Newton's method is used to find a root of  $f(x) \equiv (x - 3)^m = 0$ , for what values of  $m$  is convergence guaranteed, provided  $x_0$  is sufficiently close to the root  $r = 3$ ? Hint: Newton's method can be written in the form  $x_{n+1} = g(x_n)$ , what is  $g(x)$ ?
  
  
  
  
  
  
  
  
  
  
5. If  $a = -1000, b = 1000$  and  $f(a)$  and  $f(b)$  have opposite signs, how many bisection iterations are required to find a root between  $a$  and  $b$  to an accuracy of  $10^{-12}$ ?

