Math 4329, Test I

Name ______________________________

1. a. If $f(x) = ln(cos(x))$, find the Taylor polynomial $T_2(x)$ of degree 2 which matches f, f' and f'' at $a = 0$.

   b. Use the Taylor remainder formula to get a reasonable bound on the error $|f(x) - T_2(x)|$ at $x = 0.1$.

2. A certain computer stores floating point numbers in a 128-bit word. If a floating point number is written in normalized binary form $(1.xxxxx..._2 * 2^e)$, it is stored using one sign bit (0 if the number is positive), then $e + 4095$ is stored in binary in the next 13 bits, and then the mantissa xxxxx... is stored in the final 114 bits. Show exactly how the number -12.25 would be stored on this computer. Also: approximately how many **decimal** digits of accuracy does this machine have?

3. Compute the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $10^{-5}, 10^{-7}$ and 10^{-14}.

4. The fixed point iteration $x_{n+1} = x_n + sin(x_n)$ has roots at $r = n\pi$ for any integer n. Will this iteration converge if you start very close to the root $r = 0$? Will it converge if you start near the root $r = \pi$? In both cases, if it does converge, what is the order of convergence?

5. Show how Newton's method could be used to find $b^{\frac{1}{m}}$ for $b > 0$, where m is a positive integer, without doing anything other than add, subtract, multiply and divide.

6. Write the secant iteration for solving $f(x) = 1/x - b = 0$, in a form where no divisions are required (thus this iteration could be used to compute $1/b$ on a computer which cannot do divisions).