

## Math 4329, Test I

Name \_\_\_\_\_

1.
  - a. If  $f(x) = x^5 + 2x^2$ , find the Taylor polynomial  $T_3(x)$  of degree 3 which matches  $f, f', f''$  and  $f'''$  at  $a = 1$ .
  
  - b. Use the Taylor remainder formula to get a reasonable bound on the error  $|f(x) - T_3(x)|$  in the interval  $-0.5 \leq x \leq 1.5$ .
  
2. Write the quadratic formula root  $[-b + \sqrt{b^2 - 4ac}]/(2a)$  in a form so that there are no serious problems with roundoff error, when  $b$  is positive and very large compared to  $ac$ .
  
3. Write out (and simplify) a *secant* method iteration, used to find  $\sqrt{b}$ , which does only basic arithmetic (add, subtract, multiply and divide, no square roots).

4. If Newton's method is used to find a root of  $f(x) \equiv (x - 3)^7 = 0$ ,
- Will Newton's method converge for  $x_0$  close to the root  $r = 3$ ? Explain.
  - What is the order of convergence, if it converges?
  - Will Newton's method converge for all  $x_0$ ? Explain.
5. A certain computer stores floating point numbers in a 32-bit word, which includes 1 sign bit, 9 bits for the exponent, and 22 bits for the mantissa. *Approximately*
- What is the underflow limit (smallest positive number)?
  - What is the overflow limit (largest positive number)?
  - What is the machine precision (smallest positive number such that  $1 + \epsilon > 1$ )?
6. If the fixed point iteration  $x_{n+1} = x_n + cf(x_n)$  is used near a root  $r$  of  $f(x) = 0$ , how should the constant  $c$  be chosen in order to ensure the fastest convergence?