Math 4329, Test I

Name ______________________________

1. a. If $f(x) = e^{2x}$, find the Taylor polynomial $T_n(x)$ of degree n which matches $f, f', f''...f^{(n)}$ at $a = 0$.

   b. Use the Taylor remainder formula to get a reasonable bound (in terms of n) on the error $|f(x) - T_n(x)|$ at $x = 3$.

2. Computer A stores floating point numbers in a 96-bit word, which includes 1 sign bit, 11 bits for the exponent, and 84 bits for the mantissa. Computer B stores floating point numbers in a 96-bit word, with 1 sign bit, 25 bits for the exponent, and 70 bits for the mantissa.

   a. Which computer can handle larger numbers?

   b. Which computer has higher accuracy? Approximately how many significant **decimal** digits of accuracy does this computer have?

3. A root-finder produces approximations $x_3 = 5.01, x_4 = 5.0001, x_5 = 5.000\,000\,06$ where one root is $r = 5$? Estimate the experimental order of convergence. What method have we studied that has approximately this order?

4. Write $\frac{\sqrt{4+x}-2}{x}$ in a form where there is no serious problem with roundoff, when $x \approx 0$.

5. a. Newton's method is sometimes used to find $\frac{1}{b}$ by computing the root of $f(x) = b - \frac{1}{x}$. Write the Newton iteration in a form where no divisions are required (thus we can find $\frac{1}{b}$ without doing any divisions).

   b. Same as (5a) but use the secant method.

6. The polynomial $x^3 - x^2 - x - 1$ has one real root, at $x = 1.839$. We can write $x^3 - x^2 - x - 1 = 0$ in the form $x^3 = x^2 + x + 1$, or $x = 1 + \frac{1}{x} + \frac{1}{x^2}$ and try the iteration $x_{n+1} = 1 + \frac{1}{x_n} + \frac{1}{x_n^2}$.

   Will this converge, for x_0 near 1.839? Justify your answer without actually doing any iterations.