Math 4329, Test I

Name ______________________________

1. a. If $f(x) = e^{x/2}$ use the Taylor remainder formula to get a reasonable bound on the error $|f(x) - T_n(x)|$ for $-2 \leq x \leq 2$, where $T_n(x)$ is the Taylor polynomial of degree n for $f(x)$, at $a = 0$.

   b. Approximately how large does n need to be so that this error bound is less than 10^{-10}?

2. A certain computer stores floating point numbers in a 128-bit word, which includes 1 sign bit, 17 bits for the exponent, and 110 bits for the mantissa (significand). Assuming a normalized binary form is used $(1.xxxxx..._2 * 2^e)$ **approximately** what are:

   a. the overflow limit (largest positive number)

   b. the machine precision (smallest $\epsilon > 0$ such that $1 + \epsilon > 1$)

3. Consider that fixed-point iteration $x_{n+1} = 2.5x_n(1 - x_n)$.

   a. What are the two roots (points r such that if $x_n = r$, x_{n+1} will still equal r)?

   b. Analyze each root to determine if the iteration will converge (and if so, with what order) when you start close to that root.

4. Estimate the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $10^{-5}, 10^{-7}$ and 10^{-14}.

5. The root of $f(x) \equiv \frac{1}{x} - b = 0$ is $x = \frac{1}{b}$.

   a. Write Newton's iteration for solving $f(x) = 0$ in a form so that no divisions are required; thus providing a way to find $\frac{1}{b}$ without doing any divisions.

   b. Same problem, but use the secant iteration.