Math 4329, Test I

Name ______________________________

1. a. If $f(x) = ln(cos(x))$, find the Taylor polynomial $T_2(x)$ of degree 2 which matches f, f' and f'' at $a = 0$.

   b. Use the Taylor remainder formula to get a reasonable bound on the error $|f(x) - T_2(x)|$ in the interval $-0.01 < x < 0.01$.

2. IEEE single precision floating point numbers are stored in a 32-bit word, which includes 1 sign bit, 8 bits for the exponent, and 23 bits for the mantissa (significand). Assuming a normalized binary form is used $(1.xxxxx..._2 * 2^e)$ **approximately** what are:

   a. the overflow limit (largest positive number)

   b. the machine precision (smallest $\epsilon > 0$ such that $1 + \epsilon > 1$)

3. a. For what values of a will the iteration $x_{n+1} = x_n + a * sin(x_n)$ converge for x_0 sufficiently close to the root $r = \pi$?

   b. For what value of a will this iteration converge at least quadratically?

4. Estimate the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $0.05, 0.001$ and $0.000\ 000\ 7$.

5. $r = \frac{1}{a}$ is a root of $f(x) = \frac{1}{x} - a$. Write Newton's iteration for finding this root, in a form where no divisions are required; thus this formula can be used to find $\frac{b}{a} = b(\frac{1}{a})$ on a computer which cannot do divisions.

6. For the secant method, $e_{n+1} \approx Me_n e_{n-1}$. If the order of the secant method is α (ie, $e_{n+1} \approx Ce_n^\alpha$, and thus also $e_n \approx Ce_{n-1}^\alpha$), find α from this.