Math 4329, Test I

Name ____________________________

1. a. If $f(x) = x^3 + 2x$, find the Taylor polynomial $T_2(x)$ of degree 2 which matches f, f' and f'' at $a = 2$.

   b. Use the Taylor remainder formula to get a reasonable bound on the error $|f(x) - T_2(x)|$ in the interval $0 \le x \le 4$.

2. Computer A stores floating point numbers in a 128-bit word, which includes 1 sign bit, 31 bits for the exponent, and 96 bits for the mantissa. Computer B stores floating point numbers in a 128-bit word, with 1 sign bit, 13 bits for the exponent, and 114 bits for the mantissa.

   a. Which computer can handle larger numbers? Approximately what is the overflow limit for this computer?

   b. Which computer has higher accuracy? Approximately how many significant **decimal** digits of accuracy does this computer have?

3. If the fixed point iteration $x_{n+1} = x_n + cf(x_n)$ is used near a root r of $f(x) = 0$, how should the constant c be chosen in order to ensure the fastest convergence?

4. If Newton's method is used to find a root of $f(x) \equiv (x-3)^4 = 0$, for what values of x_0 is convergence to the root $r = 3$ guaranteed? Hint: Newton's method can be written in the form $x_{n+1} = g(x_n)$, what is $g(x)$?

5. If $a = -1000, b = 1000$ and $f(a)$ and $f(b)$ have opposite signs, how many bisection iterations are required to find a root between a and b to an accuracy of 10^{-12}?

6. A root-finder produces approximations $x_3 = 6.01, x_4 = 6.0001, x_5 = 6.000\,000\,06$ when applied to $f(x) \equiv x^2 - 36 = 0$. Estimate the experimental order of convergence. What method have we studied that has approximately this order?

7. Consider that fixed-point iteration $x_{n+1} = 2x_n(1 - x_n)$.

   a. What are the two roots (points r such that if $x_n = r$, x_{n+1} will still equal r)?

   b. Analyze each root to determine if the iteration will converge (and if so, with what order) when you start close to that root.