Math 4329, Test II

Name ____________________________

1. If

$$\begin{aligned} s(x) &= 0 && for\ 0 \le x \le 1 \\ s(x) &= A(x-1)^3 && for\ 1 \le x \le 2 \end{aligned}$$

   a. For what value(s) of A is $s(x)$ a cubic spline?

   b. For what value(s) of A is $s(x)$ a natural cubic spline?

2. Find A, B, C such that the approximation $u'(t) \approx \frac{Au(t)+Bu(t-h)+Cu(t-2h)}{h}$ is as high order as possible.

3. If $P_4(x)$ is the fourth degree polynomial that interpolates to $f(x) = sin(3x)$ at $x = 0, 0.1, 0.2, 0.3, 0.4$, find a reasonable bound on the error at $x = 0.15$.

4. True or False:

   a. If Gaussian elimination is used with NO pivoting, large roundoff errors may result even if A is well-conditioned.

   b. If Gaussian elimination is used with partial pivoting, the solution is usually very accurate even if A is ill-conditioned.

   c. The Gauss-Seidel iterative method (for $Ax = b$) is generally slower than the Jacobi method.

   d. The Jacobi iterative method (for $Ax = b$) converges only if the matrix is diagonal-dominant.

   e. A quadrature method which has $O(h^3)$ error will give a smaller error than an $O(h)$ method, for any h.

   f. Roundoff error is much more serious, in general, for derivative approximations than for integral approximations.

   g. Gaussian elimination, when applied to a general N by N linear system, requires $O(N^3)$ arithmetic operations.

   h. If $s(x)$ is a cubic spline, then s, s', s'' and s''' must be continuous everywhere.

   i. If a quadrature method is exact for all polynomials of degree n, its error is $O(h^n)$ for general smooth functions.

5. Find A, B which make the approximation

   $\int_0^h f(x)dx \approx Ahf(0.2h) + Bhf(0.9h)$

   as high order as possible.

6. Consider the linear system $Ax = b$, where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

a. Is the matrix A diagonal dominant?

b. Do 3 iterations of the Jacobi iterative method to solve $Ax = b$, starting with $(0, 0)$. Does it appear to be converging?

7. If

$$A = \begin{bmatrix} 1 & 2.00001 \\ 2 & 4 \end{bmatrix}$$

and you are using a computer with about 15 decimal digits of precision, about how many significant figures would you expect the solution of $Ax = b$ to have, for arbitrary b?