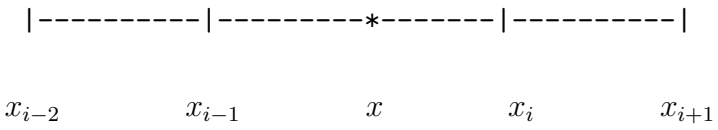
Math 4329, Test II

Name _____

1. Suppose values of $f(x) = cos(\pi x)$ are given in a table between $x = 0$ and $x = 2$ with steps of size $h = 0.001$. An estimate of $f(x)$ for x between two tabular points x_{i-1} and x_i is made by interpolating a cubic polynomial to the two points to the left x_{i-2}, x_{i-1} and the two points to the right x_i, x_{i+1}. Find as small an upper bound as you can on the interpolation error. (Don't be lazy for this problem, get the lowest bound you can.)



2. Use Taylor series expansions to determine the error in the approximation $u'''(t) \approx \frac{u(t+3h)-3u(t+2h)+3u(t+h)-u(t)}{h^3}$

3. Find A, B which make the approximation

   $\int_0^h f(x)dx \approx Ahf(0.3h) + Bhf(0.9h)$

   as high order as possible. With your choice of A, B, what is the degree of precision, and what is the order of the error (power of h that the error is proportional to) in this approximation? (Hint: this is a *local* quadrature formula, so we are asking for the *local* error, not the *global* error.)

4. Consider the linear system:

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$$

   a. Write out the equations for the Jacobi iterative method for solving this system (don't actually do any iterations).

   b. Write out the equations for the Gauss-Seidel iterative method for solving this system.

c. True or False: the Jacobi iterative method (4a) will converge for *any* starting vector (x_0, y_0, z_0). Give a reason for your answer.

d. Given that

$$A^{-1} = \begin{bmatrix} 0.1419 & -0.0878 & 0.0068 \\ 0.0135 & 0.2297 & -0.0946 \\ -0.0203 & 0.1554 & 0.1419 \end{bmatrix}$$

find the condition number of A (using L_∞ norm). If you were to solve the linear system above using Gaussian elimination with partial pivoting, would you expect serious roundoff errors?

5. Define a natural cubic spline.