

Math 4329, Test II

Name \_\_\_\_\_

1. a. A table of values for  $f(x)$  is:

$x$	$f(x)$
1.50	0.0000
1.51	0.0030
1.52	0.0000

Use quadratic interpolation to estimate  $f(1.505)$ .

- b. If it is known that  $|f'''(x)| < 0.1$  for all  $x$ , obtain a reasonable bound on the error in your estimate of  $f(1.505)$ .

2. Use Taylor series expansions to determine the error in the approximation  $u''(t) \approx \frac{u(t+2h) - 2u(t+h) + u(t)}{h^2}$

3. If

$$A = \begin{bmatrix} 2.000001 & -2 \\ -1 & 1 \end{bmatrix}$$

- a. Find the condition number of  $A$  (use  $L_\infty$  norm).
  
- b. If the machine precision is  $10^{-15}$ , estimate the relative error in the solution  $x$  when  $Ax = b$  is solved using Gauss elimination with partial pivoting.

4. Consider the linear system:

$$\begin{bmatrix} -7 & 2 & 2 \\ 0 & -3 & 1 \\ 2 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ -3 \end{bmatrix}$$

- a. Write out the equations for the Jacobi iterative method for solving this system (don't actually do any iterations).
  
  
  
  
  
  
  
  
  
  
- b. Write out the equations for the Gauss-Seidel iterative method for solving this system.
  
  
  
  
  
  
  
  
  
  
- c. True or False: the Jacobi iterative method (4a) will converge for *any* starting vector  $(x_0, y_0, z_0)$ . Give a reason for your answer.

5. The following function is a cubic spline for what values of  $a, b, c$ ?

$$\begin{aligned} s(x) &= 2x^3 - 3x^2 + 3x - 4 && \text{for } 0 < x \leq 1 \\ &= x^3 + ax^2 + bx + c && \text{for } 1 < x \leq 2 \end{aligned}$$

6. Determine values for  $A, r$  which make

$$\int_0^h f(x)dx \approx Ahf(rh) + Ahf((1-r)h)$$

as high order as possible.