Math 4329, Test II

Name ____________________________

Do problem 5 and any 3 of the first 4. Mark clearly which 3 to grade, no extra credit for doing all 4.

1. Use Taylor series expansions to determine the error in the approximation

$$u^{iv}(t) \approx \frac{u(t+2h)-4u(t+h)+6u(t)-4u(t-h)+u(t-2h)}{h^4}$$

Hint:

$u(t-2h) = u-2hu'+4h^2u''/2-8h^3u'''/6+16h^4u^{iv}/24-32h^5u^v/120+64h^6u^{vi}/720...$
$u(t-h) = u-hu'+h^2u''/2-h^3u'''/6+h^4u^{iv}/24-h^5u^v/120+h^6u^{vi}/720...$
$u(t) = u$
$u(t+h) = u+hu'+h^2u''/2+h^3u'''/6+h^4u^{iv}/24+h^5u^v/120+h^6u^{vi}/720...$
$u(t+2h) = u+2hu'+4h^2u''/2+8h^3u'''/6+16h^4u^{iv}/24+32h^5u^v/120+64h^6u^{vi}/720...$

2. If $p_N(x)$ is the polynomial of degree N which interpolates $f(x) = cos(3x)$ at $N+1$ uniformly spaced points between 0 and π, find a bound, involving only N, on $max(0 \leq x \leq \pi)|p_N(x) - f(x)|$ Will your bound go to zero as $N \rightarrow \infty$?

3. Determine the equations which must be satisfied for

$$s(x) = \begin{cases} a(x-2)^2 + b(x-1)^3 & x \leq 1 \\ c(x-2)^2 & 1 \leq x \leq 3 \\ d(x-2)^2 + e(x-3)^3 & 3 \leq x \end{cases}$$

to be a cubic spline.

4. Find A, B, C which make the approximation

$\int_0^h f(x)dx \approx Ahf(0) + Bhf(0.4h) + Chf(0.8h)$

as high order as possible.

5. True or False:

   a. If Gaussian elimination is used with NO pivoting, large roundoff errors may result even if A is well-conditioned.

   b. If Gaussian elimination is used with partial pivoting, the solution is usually very accurate even if A is ill-conditioned.

   c. The Gauss-Seidel iterative method (for $Ax = b$) is generally slower than the Jacobi method.

   d. The Jacobi iterative method (for $Ax = b$) converges only if the matrix is diagonal-dominant.

   e. A quadrature method which has $O(h^3)$ error will give a smaller error than an $O(h)$ method, for any h.

   f. Roundoff error is much more serious, in general, for derivative approximations than for integral approximations.

   g. Gaussian elimination, when applied to a general N by N linear system, requires $O(N^3)$ arithmetic operations.

   h. If $s(x)$ is a cubic spline, then s, s', s'' and s''' must be continuous everywhere.

   i. If a quadrature method is exact for all polynomials of degree n, its error is $O(h^n)$ for general smooth functions.

   j. If a matrix A has condition number 10, we expect to lose about 10 significant digits in solving $Ax = b$ with Gauss elimination and partial pivoting.