

Math 4329, Final

Name _____

1. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Start with $(1, 10, 1)$ and do 3 iterations. What is the corresponding eigenvector?

2. Reduce

$$\begin{aligned} y'' &= 3y'y - e^t z \\ z'' &= z'z - \sqrt{y} \end{aligned}$$

to a system of 4 first order equations. The right hand sides must involve only t, u_1, u_2, u_3, u_4 . The left hand sides must be u'_1, u'_2, u'_3, u'_4 respectively.

3. If the second order Taylor series method (one more term than Euler's method) is used to solve $u' = -tu^2$, write u_{n+1} in terms of h, t_n and u_n . ($t_n = nh, u_n \approx u(t_n)$)

4. Do **one** iteration of Newton's method, starting from $(0, 0)$, to solve:

$$f(x, y) = \sqrt{x+1} + xy + 3 = 0$$
$$g(x, y) = \sin(x+2y) - \ln(1+x) = 0$$

5. How should A, r be chosen to make the approximation:

$$\int_{-1}^1 f(x) dx \approx Af(-r) + Af(0) + Af(r)$$

as high degree of precision as possible?

6. True or False:
- a. Serious roundoff error can usually be traced to operations involving multiplication or division.
 - b. The experimental order of convergence is $O(h^3)$ if a quadrature rule yields errors of 0.0032 when $h = 0.01$ and 0.0002 when $h = 0.0025$.
 - c. Of all quadrature rules with n samples points per strip, the Gauss n -point formula has the highest order of accuracy.
 - d. A disadvantage of the Runge-Kutta methods is that they require several starting values.
 - e. It is easier to vary the stepsize for a Runge-Kutta method than an Adams multistep method.
 - f. Taylor series methods are not widely used by general purpose ODE solvers because they require that the user supply derivatives of $f(t, u)$.
 - g. If $f(r) = f'(r) = 0$, Newton's method will converge **quadratically** to r if x_0 is sufficiently close to the root r .
 - h. Newton's method gives the exact root (to within roundoff error) in a single step, regardless of the starting vector, if the system of equations is linear. (Assume a unique solution exists.)
 - i. Euler's method is equivalent to a first order Taylor series method.
 - j. If a matrix A has condition number 10, we expect to lose about 10 significant digits in solving $Ax = b$ with Gauss elimination and partial pivoting.

7. If $p_4(x)$ is the fourth degree polynomial which satisfies $p_4(x_i) = f(x_i)$, $i = 0, 1, 2, 3, 4$, give a formula for the error $f(x) - p_4(x)$ at an arbitrary point x .
8. Will the iteration $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$ converge to the root $\sqrt{5}$, if the starting guess is sufficiently good? **Justify** your answer.
9. A rootfinder produces consecutive root estimates of 2.01, 2.0001, 2.000001, when the exact root is $r = 2$. Estimate the order of the method.