Math 4329, Final

Name _____

1. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Start with $(1, 10, 1)$ and do 3 iterations. What is the corresponding eigenvector?

2. Reduce

$$y'' = 3y'y - e^t z$$
$$z'' = z'z - \sqrt{y}$$

to a system of 4 first order equations. The right hand sides must involve only t, u_1, u_2, u_3, u_4. The left hand sides must be u_1', u_2', u_3', u_4' respectively.

3. If the second order Taylor series method (one more term than Euler's method) is used to solve $u' = -tu^2$, write u_{n+1} in terms of h, t_n and u_n. $(t_n = nh, u_n \approx u(t_n))$

4. Do **one** iteration of Newton's method, starting from $(0, 0)$, to solve:
   $f(x, y) = \sqrt{x+1} + xy + 3 = 0$
   $g(x, y) = sin(x + 2y) - ln(1 + x) = 0$

5. How should A, r be chosen to make the approximation:

   $\int_{-1}^{1} f(x)dx \approx Af(-r) + Af(0) + Af(r)$

   as high degree of precision as possible?

6. True or False:

   a. Serious roundoff error can usually be traced to operations involving multiplication or division.

   b. The experimental order of convergence is $O(h^3)$ if a quadrature rule yields errors of 0.0032 when $h = 0.01$ and 0.0002 when $h = 0.0025$.

   c. Of all quadrature rules with n sample points per strip, the Gauss n-point formula has the highest order of accuracy.

   d. A disadvantage of the Runge-Kutta methods is that they require several starting values.

   e. It is easier to vary the stepsize for a Runge-Kutta method than an Adams multistep method.

   f. Taylor series methods are not widely used by general purpose ODE solvers because they require that the user supply derivatives of $f(t, u)$.

   g. If $f(r) = f'(r) = 0$, Newton's method will converge **quadratically** to r if x_0 is sufficiently close to the root r.

   h. Newton's method gives the exact root (to within roundoff error) in a single step, regardless of the starting vector, if the system of equations is linear. (Assume a unique solution exists.)

   i. Euler's method is equivalent to a first order Taylor series method.

   j. If a matrix A has condition number 10, we expect to lose about 10 significant digits in solving $Ax = b$ with Gauss elimination and partial pivoting.

7. If $p_4(x)$ is the fourth degree polynomial which satisfies $p_4(x_i) = f(x_i)$, $i = 0, 1, 2, 3, 4$, give a formula for the error $f(x) - p_4(x)$ at an arbitrary point x.

8. Will the iteration $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$ converge to the root $\sqrt{5}$, if the starting guess is sufficiently good? **Justify** your answer.

9. A rootfinder produces consecutive root estimates of $2.01, 2.0001, 2.000001$, when the exact root is $r = 2$. Estimate the order of the method.