Math 4329, Final

Name ______________________________

1. a. Let $T_4(x)$ be the Taylor polynomial of degree 4 which matches $f(x), f'(x), f''(x), f'''(x)$ and $f^{iv}(x)$ at $x = 1$, where $f(x) = e^{-2x}$. Find a reasonable bound on

   $max_{0 \le x \le 2} |T_4(x) - f(x)| \le$

   b. Let $L_4(x)$ be the Lagrange polynomial of degree 4 which matches $f(x)$ at $x = 0, 0.5, 1, 1.5$ and 2, where $f(x) = e^{-2x}$. Find a reasonable bound on

   $max_{0 \le x \le 2} |L_4(x) - f(x)| \le$

2. Determine the (global) order of the quadrature rule (hint: take $a = 0, b = h, N = 1$, so $x_{i-1} = 0$, to simplify)
   $\int_a^b f(x)dx \approx \sum_{i=1}^{N} \left[\frac{h}{4} f(x_{i-1}) + \frac{3h}{4} f(x_{i-1} + \frac{2h}{3}) \right]$

3. Use the power method to determine the largest eigenvalue and the associated eigenvector of

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

4. $f_1(x, y, z) = x^2 + yz + 3$
$f_2(x, y, z) = cos(z) + z$
$f_3(x, y, z) = \sqrt{y^2 + z^2} - 8$

   Do one iteration of Newton's method, to find a root of $f_1 = f_2 = f_3 = 0$, starting from $(x_0, y_0, z_0) = (1, 1, 0)$.

5. Consider the problem

   $u' = e^{t+u}$
   $u(1) = 2$

   Take one step of a second order Taylor series method (Euler = first order Taylor series method) to approximate $u(1.01)$.

6. Will the iteration $x_{n+1} = \frac{3}{4}x_n + 1/x_n^3$ converge when x_0 is sufficiently close to the root $r = \sqrt{2}$? (Justify your answer theoretically, without actually iterating the formula.)

7. a. Write the second order differential equation $u'' - 3u' - u = e^t$ as a system of two first order equations, that is, in the form:

   $u' = f(t, u, v) =$

   $v' = g(t, u, v) =$

   b. Now write out the formulas for u_{n+1}, v_{n+1} for Euler's method applied to this system of first order equations:

   $u_{n+1} =$

   $v_{n+1} =$

8. A method for approximating the solution of a differential equation produces an error at $t = 1$ of 0.02 when $h = 0.01$ and an error or 0.000002 when $h = 0.001$. Calculate the experimental order of convergence of this method.