Math 4329, Final

Name ______________________________

1. If the second order Taylor series method (one more term than Euler's method) is used to solve $u' = t^2 + u^2$, write u_{n+1} in terms of h, t_n and u_n only. ($t_n = nh, u_n \approx u(t_n)$)

2. a. Let $T_2(x)$ be the Taylor polynomial of degree 2 which matches $f(x), f'(x)$ and $f''(x)$ at $x = 0$, where $f(x) = \frac{1}{1+2x}$. Find a reasonable bound on

   $max_{-0.1 \leq x \leq 0.1} |T_2(x) - f(x)| \leq$

   b. Let $L_2(x)$ be the Lagrange polynomial of degree 2 which matches $f(x)$ at $x = -0.1, 0.0$ and 0.1, where $f(x) = \frac{1}{1+2x}$. Find a reasonable bound on

   $max_{-0.1 \leq x \leq 0.1} |L_2(x) - f(x)| \leq$

3. a. A root finder gives consecutive errors of $e_5 = 10^{-2}, e_6 = 10^{-4}, e_7 = 10^{-11}$. Estimate the order of the method.

b. A quadrature method gives an error of 10^{-7} when $h = 0.001$ and 10^{-11} when $h = 0.0001$. Estimate the order of the method.

c. A differential equation solver gives an answer $u(1) = 0.98888$ when $h = 0.1$, and $u(1) = 0.90666$ when $h = 0.01$, and $u(1) = 0.90600$ when $h = 0.001$. Estimate the order of the method.

4. Use the inverse power method to find the smallest (in absolute value) eigenvalue of A, if

$$A^{-1} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Start with $(1, 10, 1)$ and do 3 iterations.

5. Reduce

$$y'' = y'y - tz$$
$$z''' = z''z - y$$

to a system of 5 first order equations. The right hand sides must involve only $t, u_1, u_2, u_3, u_4, u_5$. The left hand sides must be $u_1', u_2', u_3', u_4', u_5'$ respectively.

6. Do one iteration of Newton's method, starting from $(0, 0)$, to solve:

$$f(x, y) = 2x^2 + y - 1 = 0$$
$$g(x, y) = -2x + y^2 + 1 = 0$$

7. Write $(1 + x)^{1/3} - 1$ in a form where there is no serious problem with roundoff, when $x \approx 0$. (Hint: $a^3 - b^3 = (a^2 + ab + b^2)(a - b)$)

8. How should A, B, r be chosen to make the approximation:

   $\int_{-1}^{1} f(x)dx \approx Af(-r) + Bf(0) + Af(r)$

   as high degree of precision as possible?

9. Will the iteration $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$ converge to the root $\sqrt{5}$, if the starting guess is sufficiently good? Justify your answer theoretically.