Math 4329, Final

Name ____________________________

1. a. Let $T_4(x)$ be the Taylor polynomial of degree 4 which matches $f(x), f'(x), f''(x), f'''(x)$ and $f^{iv}(x)$ at $x = 0$, where $f(x) = cos(x/6)$. Find the best possible bound on

   $max_{-\pi \leq x \leq \pi} |T_4(x) - f(x)| \leq$

   b. Let $L_4(x)$ be the Lagrange polynomial of degree 4 which matches $f(x)$ at $x = -\pi, -0.5, 0, 0.5$ and π, where $f(x) = cos(x/6)$. Find the best possible bound on

   $|L_4(1) - f(1)| \leq$

2. a. A root finder gives consecutive errors of $e_5 = 10^{-3}, e_6 = 10^{-5}, e_7 = 10^{-10}$. Estimate the order of the method.

   b. A quadrature method gives an error of 10^{-5} when $h = 10^{-2}$ and 10^{-9} when $h = 10^{-3}$. Estimate the order of the method.

   c. A differential equation solver gives an answer $u(1) = 2.18888$ when $h = 0.1$, and $u(1) = 2.10666$ when $h = 0.01$, and $u(1) = 2.10600$ when $h = 0.001$. Estimate the order of the method.

3. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

What is the corresponding eigenvector?

4. $x^2 + xy^3 = 9 - 3y$
$3x^2y - y^3 = 4 + 2x$

Do one iteration of Newton's method, to find a root of this system, starting from $(x_0, y_0) = (0, 0)$.

5. Will the iteration $x_{n+1} = x_n + sin(x_n)$ converge when x_0 is sufficiently close to the root $r = \pi$? If so, what is the order of convergence? (Justify your answer theoretically, without actually iterating the formula.)

6. a. Write the third order differential equation $u''' - 5u' - u = t^4$ as a system of three first order equations, that is, in the form:

   $u' = f(t, u, v, w) =$

   $v' = g(t, u, v, w) =$

   $w' = h(t, u, v, w) =$

   b. Now write out the formulas for $u_{n+1}, v_{n+1}, w_{n+1}$ for Euler's method applied to this system of first order equations:

   $u_{n+1} =$

   $v_{n+1} =$

   $w_{n+1} =$

7. If the third order Taylor series method (two more terms than Euler's method) is used to solve $u' = t(1 + u)$, write u_{n+1} in terms of h, t_n and u_n only. ($t_n = nh, u_n \approx u(t_n)$)